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**torchknufft**

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# USER GUIDE

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## ABOUT

`torchkbnufft` implements a non-uniform Fast Fourier Transform [1, 2] with Kaiser-Bessel gridding in PyTorch. The implementation is completely in Python, facilitating flexible deployment in readable code with no compilation. NUFFT functions are each wrapped as a `torch.autograd.Function`, allowing backpropagation through NUFFT operators for training neural networks.

This package was inspired in large part by the NUFFT implementation in the [Michigan Image Reconstruction Toolbox \(Matlab\)](#).





## INSTALLATION

Simple installation can be done via PyPI:

```
pip install torchkbnufft
```

`torchkbnufft` only requires `numpy`, `scipy`, and `torch` as dependencies.



## OPERATION MODES AND STAGES

The package has three major classes of NUFFT operation mode: table-based NUFFT interpolation, sparse matrix-based NUFFT interpolation, and forward/backward operators with Toeplitz-embedded FFTs [3]. Table interpolation is the standard operation mode, whereas the Toeplitz method is always the fastest for forward/backward NUFFTs. For some problems, sparse matrices may be fast. It is generally best to start with Table interpolation and then experiment with the other modes for your problem.

Sensitivity maps can be incorporated by passing them into a *KbNufft* or *KbNufftAdjoint* object. Auxiliary functions for calculating sparse interpolation matrices, density compensation functions, and Toeplitz filter kernels are also included.

For examples, see *Basic Usage*.



## REFERENCES

1. Fessler, J. A., & Sutton, B. P. (2003). Nonuniform fast Fourier transforms using min-max interpolation. *IEEE Transactions on Signal Processing*, 51(2), 560-574.
2. Beatty, P. J., Nishimura, D. G., & Pauly, J. M. (2005). Rapid gridding reconstruction with a minimal oversampling ratio. *IEEE Transactions on Medical Imaging*, 24(6), 799-808.
3. Feichtinger, H. G., Gr, K., & Strohmer, T. (1995). Efficient numerical methods in non-uniform sampling theory. *Numerische Mathematik*, 69(4), 423-440.

### 4.1 Basic Usage

`torchkbnufft` works primarily via PyTorch modules. You create a module with the properties of your imaging setup. The module will calculate a Kaiser-Bessel kernel and some interpolation parameters based on your inputs. Then, you apply the module to your data stored as PyTorch tensors. NUFFT operations are wrapped in `torch.autograd.Function` classes for backpropagation and training neural networks.

The following code loads a Shepp-Logan phantom and computes a single radial spoke of k-space data:

```
import torch
import torchkbnufft as tkbn
import numpy as np
from skimage.data import shepp_logan_phantom

x = shepp_logan_phantom().astype(np.complex)
im_size = x.shape
# convert to tensor, unsqueeze batch and coil dimension
# output size: (1, 1, ny, nx)
x = torch.tensor(x).unsqueeze(0).unsqueeze(0).to(torch.complex64)

klength = 64
ktraj = np.stack(
    (np.zeros(64), np.linspace(-np.pi, np.pi, klength))
)
# convert to tensor, unsqueeze batch dimension
# output size: (2, klength)
ktraj = torch.tensor(ktraj).to(torch.float)

nufft_ob = tkbn.KbNufft(im_size=im_size)
# outputs a (1, 1, klength) vector of k-space data
kdata = nufft_ob(x, ktraj)
```

The package also includes utilities for working with SENSE-NUFFT operators. The above code can be modified to include sensitivity maps.

```
smaps = torch.rand(1, 8, 400, 400) + 1j * torch.rand(1, 8, 400, 400)
sense_data = nufft_ob(x, ktraj, smaps=smaps.to(x))
```

This code first multiplies by the sensitivity coils in `smaps`, then computes a 64-length radial spoke for each coil. All operations are broadcast across coils, which minimizes interaction with the Python interpreter, helping computation speed.

Sparse matrices are an alternative to table interpolation. Their speed can vary, but they are a bit more accurate than standard table mode. The following code calculates sparse interpolation matrices and uses them to compute a single radial spoke of k-space data:

```
adjnufft_ob = tkbn.KbNufftAdjoint(im_size=im_size)

# precompute the sparse interpolation matrices
interp_mats = tkbn.calc_tensor_spmatrix(
    ktraj,
    im_size=im_size
)

# use sparse matrices in adjoint
image = adjnufft_ob(kdata, ktraj, interp_mats)
```

Sparse matrix multiplication is only implemented for real numbers in PyTorch, which can limit their speed.

The package includes routines for calculating [embedded Toeplitz kernels](#) and using them as FFT filters for the forward/backward NUFFT operations. This is very useful for gradient descent algorithms that must use the forward/backward ops in calculating the gradient. The following code shows an example:

```
adjnufft_ob = tkbn.KbNufftAdjoint(im_size=im_size)
toep_ob = tkbn.ToepNufft()

# precompute the embedded Toeplitz FFT kernel
kernel = tkbn.calc_toeplitz_kernel(ktraj, im_size)

# use FFT kernel from embedded Toeplitz matrix
image = toep_ob(image, kernel)
```

All of the examples included in this repository can be run on the GPU by sending the NUFFT object and data to the GPU prior to the function call, e.g.,

```
adjnufft_ob = adjnufft_ob.to(torch.device('cuda'))
kdata = kdata.to(torch.device('cuda'))
ktraj = ktraj.to(torch.device('cuda'))

image = adjnufft_ob(kdata, ktraj)
```

Similar to programming low-level code, PyTorch will throw errors if the underlying `dtype` and `device` of all objects are not matching. Be sure to make sure your data and NUFFT objects are on the right device and in the right format to avoid these errors.

For more details, please examine the API in [torchkbnufft](#) or the [notebooks](#) in the GitHub repository.

## 4.2 torchkbnufft

### 4.2.1 NUFFT Modules

These are the primary workhorse modules for applying NUFFT operations.

<i>KbInterp</i>	Non-uniform Kaiser-Bessel interpolation layer.
<i>KbInterpAdjoint</i>	Non-uniform Kaiser-Bessel interpolation adjoint layer.
<i>KbNufft</i>	Non-uniform FFT layer.
<i>KbNufftAdjoint</i>	Non-uniform FFT adjoint layer.
<i>ToepNufft</i>	Forward/backward NUFFT with Toeplitz embedding.

### KbInterp

**class KbInterp** (*im\_size*, *grid\_size=None*, *numpoints=6*, *n\_shift=None*, *table\_oversamp=1024*, *kb\_width=2.34*, *order=0.0*, *dtype=None*, *device=None*)  
Non-uniform Kaiser-Bessel interpolation layer.

This object interpolates a grid of Fourier data to off-grid locations using a Kaiser-Bessel kernel. Mathematically, in one dimension it estimates  $Y_m, m \in [0, \dots, M - 1]$  at frequency locations  $\omega_m$  from  $X_k, k \in [0, \dots, K - 1]$ , the oversampled DFT of  $x_n, n \in [0, \dots, N - 1]$ . To perform the estimate, this layer applies

$$Y_m = \sum_{j=1}^J X_{\{k_m+j\}_K} u_j^*(\omega_m),$$

where  $u$  is the Kaiser-Bessel kernel,  $k_m$  is the index of the root offset of nearest samples of  $X$  to frequency location  $\omega_m$ , and  $J$  is the number of nearest neighbors to use from  $X_k$ . Multiple dimensions are handled separately. For a detailed description of the notation see [Nonuniform fast Fourier transforms using min-max interpolation \(JA Fessler and BP Sutton\)](#).

When called, the parameters of this class define properties of the kernel and how the interpolation is applied.

- `im_size` is the size of the base image, analogous to  $N$  (used for calculating the kernel but not for the actual operation).
- `grid_size` is the size of the grid prior to interpolation, analogous to  $K$ . To reduce errors, NUFFT operations are done on an oversampled grid to reduce interpolation distances. This will typically be 1.25 to 2 times `im_size`.
- `numpoints` is the number of nearest to use for interpolation, i.e.,  $J$ .
- `n_shift` is the FFT shift distance, typically `im_size // 2`.

#### Parameters

- **`im_size`** (`Sequence[int]`) – Size of image with length being the number of dimensions.
- **`grid_size`** (`Optional[Sequence[int]]`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`. Default: `2 * im_size`
- **`numpoints`** (`Union[int, Sequence[int]]`) – Number of neighbors to use for interpolation in each dimension.
- **`n_shift`** (`Optional[Sequence[int]]`) – Size for `fftshift`. Default: `im_size // 2`.
- **`table_oversamp`** (`Union[int, Sequence[int]]`) – Table oversampling factor.

- **kbwidth** (`float`) – Size of Kaiser-Bessel kernel.
- **order** (`Union[float, Sequence[float]]`) – Order of Kaiser-Bessel kernel.
- **dtype** (`Optional[dtype]`) – Data type for tensor buffers. Default: `torch.get_default_dtype()`
- **device** (`Optional[device]`) – Which device to create tensors on. Default: `torch.device('cpu')`

## Examples

```
>>> image = torch.randn(1, 1, 8, 8) + 1j * torch.randn(1, 1, 8, 8)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> kb_ob = tkbn.KbInterp(im_size=(8, 8), grid_size=(8, 8))
>>> data = kb_ob(image, omega)
```

**forward** (*image*, *omega*, *interp\_mats=None*)

Interpolate from gridded data to scattered data.

Input tensors should be of shape  $(N, C) + \text{grid\_size}$ , where  $N$  is the batch size and  $C$  is the number of sensitivity coils. *omega*, the k-space trajectory, should be of size  $(\text{len}(\text{grid\_size}), \text{klength})$ , where *klength* is the length of the k-space trajectory.

If your tensors are real-valued, ensure that 2 is the size of the last dimension.

### Parameters

- **image** (`Tensor`) – Gridded data to be interpolated to scattered data.
- **omega** (`Tensor`) – k-space trajectory (in radians/voxel).
- **interp\_mats** (`Optional[Tuple[Tensor, Tensor]]`) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix KB interpolation (overrides default table interpolation).

**Return type** `Tensor`

**Returns** *image* calculated at Fourier frequencies specified by *omega*.

## KbInterpAdjoint

**class** `KbInterpAdjoint` (*im\_size*, *grid\_size=None*, *numpoints=6*, *n\_shift=None*, *table\_oversamp=1024*, *kbwidth=2.34*, *order=0.0*, *dtype=None*, *device=None*)

Non-uniform Kaiser-Bessel interpolation adjoint layer.

This object interpolates off-grid Fourier data to on-grid locations using a Kaiser-Bessel kernel. Mathematically, in one dimension it estimates  $X_k, k \in [0, \dots, K - 1]$ , the oversampled DFT of  $x_n, n \in [0, \dots, N - 1]$ , from a signal  $Y_m, m \in [0, \dots, M - 1]$  at frequency locations  $\omega_m$ . To perform the estimate, this layer applies

$$X_k = \sum_{j=1}^J \sum_{m=0}^{M-1} Y_m u_j(\omega_m) \mathbb{1}_{\{\{k_m+j\}_K=k\}},$$

where  $u$  is the Kaiser-Bessel kernel,  $k_m$  is the index of the root offset of nearest samples of  $X$  to frequency location  $\omega_m$ ,  $\mathbb{1}$  is an indicator function, and  $J$  is the number of nearest neighbors to use from  $X_k$ . Multiple dimensions are handled separably. For a detailed description of the notation see [Nonuniform fast Fourier transforms using min-max interpolation \(JA Fessler and BP Sutton\)](#).



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**Note:** This function is not the inverse of *KbInterp*; it is the adjoint.

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When called, the parameters of this class define properties of the kernel and how the interpolation is applied.

- `im_size` is the size of the base image, analogous to  $N$  (used for calculating the kernel but not for the actual operation).
- `grid_size` is the size of the grid after adjoint interpolation, analogous to  $K$ . To reduce errors, NUFFT operations are done on an oversampled grid to reduce interpolation distances. This will typically be 1.25 to 2 times `im_size`.
- `numpoints` is the number of nearest neighbors to use for interpolation, i.e.,  $J$ .
- `n_shift` is the FFT shift distance, typically `im_size // 2`.

### Parameters

- **`im_size`** (`Sequence[int]`) – Size of image with length being the number of dimensions.
- **`grid_size`** (`Optional[Sequence[int]]`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`. Default: `2 * im_size`
- **`numpoints`** (`Union[int, Sequence[int]]`) – Number of neighbors to use for interpolation in each dimension.
- **`n_shift`** (`Optional[Sequence[int]]`) – Size for `fftshift`. Default: `im_size // 2`.
- **`table_oversamp`** (`Union[int, Sequence[int]]`) – Table oversampling factor.
- **`kbwidth`** (`float`) – Size of Kaiser-Bessel kernel.
- **`order`** (`Union[float, Sequence[float]]`) – Order of Kaiser-Bessel kernel.
- **`dtype`** (`Optional[dtype]`) – Data type for tensor buffers. Default: `torch.get_default_dtype()`
- **`device`** (`Optional[device]`) – Which device to create tensors on. Default: `torch.device('cpu')`

### Examples

```
>>> data = torch.randn(1, 1, 12) + 1j * torch.randn(1, 1, 12)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> adjkb_ob = tkbn.KbInterpAdjoint(im_size=(8, 8), grid_size=(8, 8))
>>> image = adjkb_ob(data, omega)
```

**forward** (`data`, `omega`, `interp_mats=None`, `grid_size=None`)

Interpolate from scattered data to gridded data.

Input tensors should be of shape  $(N, C) + \text{klength}$ , where  $N$  is the batch size and  $C$  is the number of sensitivity coils. `omega`, the k-space trajectory, should be of size  $(\text{len}(\text{im\_size}), \text{klength})$ , where `klength` is the length of the k-space trajectory.

If your tensors are real-valued, ensure that 2 is the size of the last dimension.

### Parameters

- **`data`** (`Tensor`) – Data to be gridded.
- **`omega`** (`Tensor`) – k-space trajectory (in radians/voxel).

- **interp\_mats** (Optional[Tuple[Tensor, Tensor]]) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix KB interpolation (overrides default table interpolation).

**Return type** Tensor

**Returns** data interpolated to the grid.

## KbNufft

**class KbNufft** (*im\_size*, *grid\_size=None*, *numpoints=6*, *n\_shift=None*, *table\_oversamp=1024*, *kb\_width=2.34*, *order=0.0*, *dtype=None*, *device=None*)  
 Non-uniform FFT layer.

This object applies the FFT and interpolates a grid of Fourier data to off-grid locations using a Kaiser-Bessel kernel. Mathematically, in one dimension it estimates  $Y_m, m \in [0, \dots, M - 1]$  at frequency locations  $\omega_m$  from  $X_k, k \in [0, \dots, K - 1]$ , the oversampled DFT of  $x_n, n \in [0, \dots, N - 1]$ . To perform the estimate, this layer applies

$$X_k = \sum_{n=0}^{N-1} s_n x_n e^{-i\gamma kn}$$

$$Y_m = \sum_{j=1}^J X_{\{k_m+j\}_K} u_j^*(\omega_m)$$

In the first step, an image-domain signal  $x_n$  is converted to a gridded, oversampled frequency-domain signal,  $X_k$ . The scaling coefficients  $s_n$  are multiplied to precompensate for NUFFT interpolation errors. The oversampling coefficient is  $\gamma = 2\pi/K, K \geq N$ .

In the second step,  $u$ , the Kaiser-Bessel kernel, is used to estimate  $X_k$  at off-grid frequency locations  $\omega_m$ .  $k_m$  is the index of the root offset of nearest samples of  $X$  to frequency location  $\omega_m$ , and  $J$  is the number of nearest neighbors to use from  $X_k$ . Multiple dimensions are handled separately. For a detailed description see [Nonuniform fast Fourier transforms using min-max interpolation \(JA Fessler and BP Sutton\)](#).

When called, the parameters of this class define properties of the kernel and how the interpolation is applied.

- *im\_size* is the size of the base image, analagous to  $N$ .
- *grid\_size* is the size of the grid after forward FFT, analogous to  $K$ . To reduce errors, NUFFT operations are done on an oversampled grid to reduce interpolation distances. This will typically be 1.25 to 2 times *im\_size*.
- *numpoints* is the number of nearest neighbors to use for interpolation, i.e.,  $J$ .
- *n\_shift* is the FFT shift distance, typically *im\_size* // 2.

### Parameters

- **im\_size** (Sequence[int]) – Size of image with length being the number of dimensions.
- **grid\_size** (Optional[Sequence[int]]) – Size of grid to use for interpolation, typically 1.25 to 2 times *im\_size*. Default:  $2 * \text{im\_size}$
- **numpoints** (Union[int, Sequence[int]]) – Number of neighbors to use for interpolation in each dimension.
- **n\_shift** (Optional[Sequence[int]]) – Size for *fftshift*. Default: *im\_size* // 2.
- **table\_oversamp** (Union[int, Sequence[int]]) – Table oversampling factor.

- **kbwidth** (`float`) – Size of Kaiser-Bessel kernel.
- **order** (`Union[float, Sequence[float]]`) – Order of Kaiser-Bessel kernel.
- **dtype** (`Optional[dtype]`) – Data type for tensor buffers. Default: `torch.get_default_dtype()`
- **device** (`Optional[device]`) – Which device to create tensors on. Default: `torch.device('cpu')`

## Examples

```
>>> image = torch.randn(1, 1, 8, 8) + 1j * torch.randn(1, 1, 8, 8)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> kb_ob = tkbn.KbNufft(im_size=(8, 8))
>>> data = kb_ob(image, omega)
```

**forward** (*image, omega, interp\_mats=None, smaps=None, norm=None*)

Apply FFT and interpolate from gridded data to scattered data.

Input tensors should be of shape  $(N, C) + \text{im\_size}$ , where  $N$  is the batch size and  $C$  is the number of sensitivity coils. `omega`, the k-space trajectory, should be of size  $(\text{len}(\text{im\_size}), \text{klength})$ , where `klength` is the length of the k-space trajectory.

If your tensors are real, ensure that 2 is the size of the last dimension.

### Parameters

- **image** (`Tensor`) – Object to calculate off-grid Fourier samples from.
- **omega** (`Tensor`) – k-space trajectory (in radians/voxel).
- **interp\_mats** (`Optional[Tuple[Tensor, Tensor]]`) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix NUFFT interpolation (overrides default table interpolation).
- **smaps** (`Optional[Tensor]`) – Sensitivity maps. If input, these will be multiplied before the forward NUFFT.
- **norm** (`Optional[str]`) – Whether to apply normalization with the FFT operation. Options are "ortho" or `None`.

**Return type** `Tensor`

**Returns** `image` calculated at Fourier frequencies specified by `omega`.

## KbNufftAdjoint

**class KbNufftAdjoint** (*im\_size, grid\_size=None, numpoints=6, n\_shift=None, table\_oversamp=1024, kbwidth=2.34, order=0.0, dtype=None, device=None*)

Non-uniform FFT adjoint layer.

This object interpolates off-grid Fourier data to on-grid locations using a Kaiser-Bessel kernel prior to inverse DFT. Mathematically, in one dimension it estimates  $x_n, n \in [0, \dots, N - 1]$  from a off-grid signal  $Y_m, m \in [0, \dots, M - 1]$  where the off-grid frequency locations are  $\omega_m$ . To perform the estimate, this layer applies

$$X_k = \sum_{j=1}^J \sum_{m=0}^{M-1} Y_m u_j(\omega_m) \mathbb{1}_{\{\{k_m+j\}_K=k\}},$$

$$x_n = s_n^* \sum_{k=0}^{K-1} X_k e^{i\gamma kn}$$

In the first step,  $u$ , the Kaiser-Bessel kernel, is used to estimate  $Y$  at on-grid frequency locations from locations at  $\omega$ .  $k_m$  is the index of the root offset of nearest samples of  $X$  to frequency location  $\omega_m$ ,  $\mathcal{I}$  is an indicator function, and  $J$  is the number of nearest neighbors to use from  $X_k, k \in [0, \dots, K - 1]$ .

In the second step, an image-domain signal  $x_n$  is estimated from a gridded, oversampled frequency-domain signal,  $X_k$  by applying the inverse FFT, after which the complex conjugate scaling coefficients  $s_n$  are multiplied. The oversampling coefficient is  $\gamma = 2\pi/K, K \geq N$ . Multiple dimensions are handled separately. For a detailed description see [Nonuniform fast Fourier transforms using min-max interpolation \(JA Fessler and BP Sutton\)](#).

---

**Note:** This function is not the inverse of *KbNufft*; it is the adjoint.

---

When called, the parameters of this class define properties of the kernel and how the interpolation is applied.

- `im_size` is the size of the base image, analagous to  $N$ .
- `grid_size` is the size of the grid after adjoint interpolation, analogous to  $K$ . To reduce errors, NUFFT operations are done on an oversampled grid to reduce interpolation distances. This will typically be 1.25 to 2 times `im_size`.
- `numpoints` is the number of nearest neighbors to use for interpolation, i.e.,  $J$ .
- `n_shift` is the FFT shift distance, typically `im_size // 2`.

#### Parameters

- `im_size` (`Sequence[int]`) – Size of image with length being the number of dimensions.
- `grid_size` (`Optional[Sequence[int]]`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`. Default: `2 * im_size`
- `numpoints` (`Union[int, Sequence[int]]`) – Number of neighbors to use for interpolation in each dimension.
- `n_shift` (`Optional[Sequence[int]]`) – Size for `fftshift`. Default: `im_size // 2`.
- `table_oversamp` (`Union[int, Sequence[int]]`) – Table oversampling factor.
- `kbwidth` (`float`) – Size of Kaiser-Bessel kernel.
- `order` (`Union[float, Sequence[float]]`) – Order of Kaiser-Bessel kernel.
- `dtype` (`Optional[dtype]`) – Data type for tensor buffers. Default: `torch.get_default_dtype()`
- `device` (`Optional[device]`) – Which device to create tensors on. Default: `torch.device('cpu')`

## Examples

```
>>> data = torch.randn(1, 1, 12) + 1j * torch.randn(1, 1, 12)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> adjkb_ob = tkbn.KbNufftAdjoint(im_size=(8, 8))
>>> image = adjkb_ob(data, omega)
```

**forward** (*data*, *omega*, *interp\_mats=None*, *smaps=None*, *norm=None*)

Interpolate from scattered data to gridded data and then iFFT.

Input tensors should be of shape  $(N, C) + \text{klength}$ , where  $N$  is the batch size and  $C$  is the number of sensitivity coils. *omega*, the k-space trajectory, should be of size  $(\text{len}(\text{im\_size}), \text{klength})$ , where *klength* is the length of the k-space trajectory.

If your tensors are real, ensure that 2 is the size of the last dimension.

### Parameters

- **data** (*Tensor*) – Data to be gridded and then inverse FFT'd.
- **omega** (*Tensor*) – k-space trajectory (in radians/voxel).
- **interp\_mats** (*Optional[Tuple[Tensor, Tensor]]*) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix NUFFT interpolation (overrides default table interpolation).
- **smaps** (*Optional[Tensor]*) – Sensitivity maps. If input, these will be multiplied before the forward NUFFT.
- **norm** (*Optional[str]*) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** *Tensor*

**Returns** *data* transformed to the image domain.

## ToeplNufft

**class ToeplNufft**

Forward/backward NUFFT with Toeplitz embedding.

This module applies  $Tx$ , where  $T$  is a matrix such that  $T \approx A'A$ , where  $A$  is a NUFFT matrix. Using Toeplitz embedding, this module approximates the  $A'A$  operation without interpolations, which is extremely fast.

The module is intended to be used in combination with an FFT kernel computed as frequency response of an embedded Toeplitz matrix. You can use `calc_toeplitz_kernel()` to calculate the kernel.

The FFT kernel should be passed to this module's forward operation, which applies a (zero-padded) FFT filter using the kernel.

## Examples

```
>>> image = torch.randn(1, 1, 8, 8) + 1j * torch.randn(1, 1, 8, 8)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> toep_ob = tkbn.ToepNufft()
>>> kernel = tkbn.calc_toeplitz_kernel(omega, im_size=(8, 8))
>>> image = toep_ob(image, kernel)
```

**forward** (*image*, *kernel*, *smaps=None*, *norm='ortho'*)  
Toeplitz NUFFT forward function.

### Parameters

- **image** (`Tensor`) – The image to apply the forward/backward Toeplitz-embedded NUFFT to.
- **kernel** (`Tensor`) – The filter response taking into account Toeplitz embedding.
- **norm** (`Optional[str]`) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** `Tensor`

**Returns** *image* after applying the Toeplitz forward/backward NUFFT.

## 4.2.2 Utility Functions

Functions for calculating density compensation and Toeplitz kernels.

<code>calc_density_compensation_function</code>	Numerical density compensation estimation.
<code>calc_tensor_spmatrix</code>	Builds a sparse matrix for interpolation.
<code>calc_toeplitz_kernel</code>	Calculates an FFT kernel for Toeplitz embedding.

### calc\_density\_compensation\_function

**calc\_density\_compensation\_function** (*ktraj*, *im\_size*, *num\_iterations=10*, *grid\_size=None*, *numpoints=6*, *n\_shift=None*, *table\_oversamp=1024*, *kbwidth=2.34*, *order=0.0*)

Numerical density compensation estimation.

This function has optional parameters for initializing a NUFFT object. See *KbInterp* for details.

- *ktraj* should be of size  $(\text{len}(\text{im\_size}), \text{klength})$ , where *klength* is the length of the k-space trajectory.

Based on the [method of Pipe](#).

This code was contributed by Chaithya G.R.

### Parameters

- **ktraj** (`Tensor`) – k-space trajectory (in radians/voxel).
- **im\_size** (`Sequence[int]`) – Size of image with length being the number of dimensions.
- **num\_iterations** (`int`) – Number of iterations.
- **grid\_size** (`Optional[Sequence[int]]`) – Size of grid to use for interpolation, typically 1.25 to 2 times *im\_size*. Default:  $2 * \text{im\_size}$

- **numpoints** (`Union[int, Sequence[int]]`) – Number of neighbors to use for interpolation in each dimension. Default: 6
- **n\_shift** (`Optional[Sequence[int]]`) – Size for fftshift. Default: `im_size // 2`.
- **table\_oversamp** (`Union[int, Sequence[int]]`) – Table oversampling factor.
- **kbwidth** (`float`) – Size of Kaiser-Bessel kernel.
- **order** (`Union[float, Sequence[float]]`) – Order of Kaiser-Bessel kernel.

**Return type** `Tensor`

**Returns** The density compensation coefficients for `ktraj`.

## Examples

```
>>> data = torch.randn(1, 1, 12) + 1j * torch.randn(1, 1, 12)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> dcomp = tkbn.calc_density_compensation_function(omega, (8, 8))
>>> adjkb_ob = tkbn.KbNufftAdjoint(im_size=(8, 8))
>>> image = adjkb_ob(data * dcomp, omega)
```

## calc\_tensor\_spmatrix

**calc\_tensor\_spmatrix** (`omega`, `im_size`, `grid_size=None`, `numpoints=6`, `n_shift=None`, `table_oversamp=1024`, `kbwidth=2.34`, `order=0.0`)

Builds a sparse matrix for interpolation.

This builds the interpolation matrices directly from scipy Kaiser-Bessel functions, so using them for a NUFFT should be a little more accurate than table interpolation.

This function has optional parameters for initializing a NUFFT object. See `KbNufft` for details.

- `omega` should be of size `(len(im_size), klength)`, where `klength` is the length of the k-space trajectory.

### Parameters

- **omega** (`Tensor`) – k-space trajectory (in radians/voxel).
- **im\_size** (`Sequence[int]`) – Size of image with length being the number of dimensions.
- **grid\_size** (`Optional[Sequence[int]]`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`. Default: `2 * im_size`
- **numpoints** (`Union[int, Sequence[int]]`) – Number of neighbors to use for interpolation in each dimension.
- **n\_shift** (`Optional[Sequence[int]]`) – Size for fftshift. Default: `im_size // 2`.
- **table\_oversamp** (`Union[int, Sequence[int]]`) – Table oversampling factor.
- **kbwidth** (`float`) – Size of Kaiser-Bessel kernel.
- **order** (`Union[float, Sequence[float]]`) – Order of Kaiser-Bessel kernel.
- **dtype** – Data type for tensor buffers. Default: `torch.get_default_dtype()`
- **device** – Which device to create tensors on. Default: `torch.device('cpu')`

**Return type** `Tuple[Tensor, Tensor]`

**Returns** 2-Tuple of (real, imaginary) tensors for NUFFT interpolation.

### Examples

```
>>> data = torch.randn(1, 1, 12) + 1j * torch.randn(1, 1, 12)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> spmats = tkbn.calc_tensor_spmatrix(omega, (8, 8))
>>> adjkb_ob = tkbn.KbNufftAdjoint(im_size=(8, 8))
>>> image = adjkb_ob(data, omega, spmats)
```

### calc\_toeplitz\_kernel

**calc\_toeplitz\_kernel** (*omega*, *im\_size*, *weights=None*, *norm=None*, *grid\_size=None*, *numpoints=6*, *n\_shift=None*, *table\_oversamp=1024*, *kbwidth=2.34*, *order=0.0*)

Calculates an FFT kernel for Toeplitz embedding.

The kernel is calculated using an adjoint NUFFT object. If the adjoint applies  $A'$ , then this script calculates  $D$  where  $F'DF \approx A'WA$ , where  $F$  is a DFT matrix and  $W$  is a set of non-Cartesian k-space weights.  $D$  can then be used to approximate  $A'WA$  without any interpolation operations.

For details on Toeplitz embedding, see [Efficient numerical methods in non-uniform sampling theory](#) (Feichtinger et al.).

This function has optional parameters for initializing a NUFFT object. See [KbNufftAdjoint](#) for details.

---

**Note:** This function is intended to be used in conjunction with [ToepNufft](#) for forward operations.

---

- *omega* should be of size  $(\text{len}(\text{im\_size}), \text{klength})$ , where *klength* is the length of the k-space trajectory.

#### Parameters

- **omega** ([Tensor](#)) – k-space trajectory (in radians/voxel).
- **im\_size** ([Sequence\[int\]](#)) – Size of image with length being the number of dimensions.
- **weights** ([Optional\[Tensor\]](#)) – Non-Cartesian k-space weights (e.g., density compensation). Default: `torch.ones(omega.shape[1])`
- **norm** ([Optional\[str\]](#)) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.
- **grid\_size** ([Optional\[Sequence\[int\]\]](#)) – Size of grid to use for interpolation, typically 1.25 to 2 times *im\_size*. Default: `2 * im_size`
- **numpoints** ([Union\[int, Sequence\[int\]\]](#)) – Number of neighbors to use for interpolation in each dimension.
- **n\_shift** ([Optional\[Sequence\[int\]\]](#)) – Size for fftshift. Default: `im_size // 2`.
- **table\_oversamp** ([Union\[int, Sequence\[int\]\]](#)) – Table oversampling factor.
- **kbwidth** ([float](#)) – Size of Kaiser-Bessel kernel.
- **order** ([Union\[float, Sequence\[float\]\]](#)) – Order of Kaiser-Bessel kernel.

**Return type** [Tensor](#)



**Returns** The FFT kernel for approximating the forward/adjoint operation.

### Examples

```
>>> image = torch.randn(1, 1, 8, 8) + 1j * torch.randn(1, 1, 8, 8)
>>> omega = torch.rand(2, 12) * 2 * np.pi - np.pi
>>> toep_ob = tkbn.ToepNufft()
>>> kernel = tkbn.calc_toeplitz_kernel(omega, im_size=(8, 8))
>>> image = toep_ob(image, kernel)
```

## 4.2.3 Math Functions

Complex mathematical operations (gradually being removed as of PyTorch 1.7).

<i>absolute</i>	Complex absolute value.
<i>complex_mult</i>	Complex multiplication.
<i>complex_sign</i>	Complex sign function value.
<i>conj_complex_mult</i>	Complex multiplication, conjugating second input.
<i>imag_exp</i>	Imaginary exponential.
<i>inner_product</i>	Complex inner product.

### torchkbnufft.absolute

**absolute** (*val*, *dim=-1*)  
Complex absolute value.

#### Parameters

- **val** (*Tensor*) – A tensor to have its absolute value computed.
- **dim** (*int*) – An integer indicating the complex dimension (for real inputs only).

**Return type** *Tensor*

**Returns** The absolute value of *val*.

### torchkbnufft.complex\_mult

**complex\_mult** (*val1*, *val2*, *dim=-1*)  
Complex multiplication.

#### Parameters

- **val1** (*Tensor*) – A tensor to be multiplied.
- **val2** (*Tensor*) – A second tensor to be multiplied.
- **dim** (*int*) – An integer indicating the complex dimension (for real inputs only).

**Return type** *Tensor*

**Returns** *val1* \* *val2*, where \* executes complex multiplication.

### torchkbnufft.complex\_sign

**complex\_sign** (*val*, *dim=-1*)  
Complex sign function value.

**Parameters**

- **val** (*Tensor*) – A tensor to have its complex sign computed.
- **dim** (*int*) – An integer indicating the complex dimension (for real inputs only).

**Return type** *Tensor*

**Returns** The complex sign of *val*.

### torchkbnufft.conj\_complex\_mult

**conj\_complex\_mult** (*val1*, *val2*, *dim=-1*)  
Complex multiplication, conjugating second input.

**Parameters**

- **val1** (*Tensor*) – A tensor to be multiplied.
- **val2** (*Tensor*) – A second tensor to be conjugated then multiplied.
- **dim** (*int*) – An integer indicating the complex dimension (for real inputs only).

**Return type** *Tensor*

**Returns**  $val3 = val1 * conj(val2)$ , where  $*$  executes complex multiplication.

### torchkbnufft.imag\_exp

**imag\_exp** (*val*, *dim=-1*, *return\_complex=True*)  
Imaginary exponential.

**Parameters**

- **val** (*Tensor*) – A tensor to be exponentiated.
- **dim** (*int*) – An integer indicating the complex dimension of the output (for real outputs only).

**Return type** *Tensor*

**Returns**  $val2 = exp(i*val)$ , where  $i$  is  $\sqrt{-1}$ .

### torchkbnufft.inner\_product

**inner\_product** (*val1*, *val2*, *dim=-1*)  
Complex inner product.

**Parameters**

- **val1** (*Tensor*) – A tensor for the inner product.
- **val2** (*Tensor*) – A second tensor for the inner product.
- **dim** (*int*) – An integer indicating the complex dimension (for real inputs only).

**Return type** *Tensor*

**Returns** The complex inner product of `val1` and `val2`.

## 4.3 torchkbnufft.functional

### 4.3.1 fft\_filter

**fft\_filter** (*image*, *kernel*, *norm='ortho'*)  
FFT-based filtering on an oversampled grid.

This is a wrapper for the operation

$$\text{output} = iFFT(\text{kernel} * FFT(\text{image}))$$

where *iFFT* and *FFT* are both implemented as oversampled FFTs.

#### Parameters

- **image** (`Tensor`) – The image to be filtered.
- **kernel** (`Tensor`) – FFT-domain filter.
- **norm** (`Optional[str]`) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** `Tensor`

**Returns** Filtered version of `image`.

## Interpolation Functions

### 4.3.2 kb\_spmat\_interp

**kb\_spmat\_interp** (*image*, *interp\_mats*)  
Kaiser-Bessel sparse matrix interpolation.

See *KbInterp* for an overall description of interpolation.

To calculate the sparse matrix tuple, see *calc\_tensor\_spmatrix()*.

#### Parameters

- **image** (`Tensor`) – Gridded data to be interpolated to scattered data.
- **interp\_mats** (`Tuple[Tensor, Tensor]`) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix KB interpolation.

**Return type** `Tensor`

**Returns** `image` calculated at scattered locations.

### 4.3.3 kb\_spmat\_interp\_adjoint

**kb\_spmat\_interp\_adjoint** (*data, interp\_mats, grid\_size*)

Kaiser-Bessel sparse matrix interpolation adjoint.

See *KbInterpAdjoint* for an overall description of adjoint interpolation.

To calculate the sparse matrix tuple, see *calc\_tensor\_spmatrix()*.

#### Parameters

- **data** (*Tensor*) – Scattered data to be interpolated to gridded data.
- **interp\_mats** (*Tuple[Tensor, Tensor]*) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix KB interpolation.

**Return type** *Tensor*

**Returns** *data* calculated at gridded locations.

### 4.3.4 kb\_table\_interp

**kb\_table\_interp** (*image, omega, tables, n\_shift, numpoints, table\_oversamp, offsets*)

Kaiser-Bessel table interpolation.

See *KbInterp* for an overall description of interpolation and how to construct the function arguments.

#### Parameters

- **image** (*Tensor*) – Gridded data to be interpolated to scattered data.
- **omega** (*Tensor*) – k-space trajectory (in radians/voxel).
- **tables** (*List[Tensor]*) – Interpolation tables (one table for each dimension).
- **n\_shift** (*Tensor*) – Size for fftshift, usually *im\_size // 2*.
- **numpoints** (*Tensor*) – Number of neighbors to use for interpolation.
- **table\_oversamp** (*Tensor*) – Table oversampling factor.
- **offsets** (*Tensor*) – A list of offsets, looping over all possible combinations of *numpoints*.

**Return type** *Tensor*

**Returns** *image* calculated at scattered locations.

### 4.3.5 kb\_table\_interp\_adjoint

**kb\_table\_interp\_adjoint** (*data, omega, tables, n\_shift, numpoints, table\_oversamp, offsets, grid\_size*)

Kaiser-Bessel table interpolation adjoint.

See *KbInterpAdjoint* for an overall description of adjoint interpolation.

#### Parameters

- **data** (*Tensor*) – Scattered data to be interpolated to gridded data.
- **omega** (*Tensor*) – k-space trajectory (in radians/voxel).
- **tables** (*List[Tensor]*) – Interpolation tables (one table for each dimension).
- **n\_shift** (*Tensor*) – Size for fftshift, usually *im\_size // 2*.

- **numpoints** (`Tensor`) – Number of neighbors to use for interpolation.
- **table\_oversamp** (`Tensor`) – Table oversampling factor.
- **offsets** (`Tensor`) – A list of offsets, looping over all possible combinations of `numpoints`.
- **grid\_size** (`Tensor`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`.

**Return type** `Tensor`

**Returns** data calculated at gridded locations.

## NUFFT Functions

### 4.3.6 kb\_spmat\_nufft

**kb\_spmat\_nufft** (*image, scaling\_coef, im\_size, grid\_size, interp\_mats, norm=None*)

Kaiser-Bessel NUFFT with sparse matrix interpolation.

See *KbNufft* for an overall description of the forward NUFFT.

To calculate the sparse matrix tuple, see *calc\_tensor\_spmatrix()*.

#### Parameters

- **image** (`Tensor`) – Image to be NUFFT'd to scattered data.
- **scaling\_coef** (`Tensor`) – Image-domain coefficients to pre-compensate for interpolation errors.
- **im\_size** (`Tensor`) – Size of image with length being the number of dimensions.
- **grid\_size** (`Tensor`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`.
- **interp\_mats** (`Tuple[Tensor, Tensor]`) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix KB interpolation.
- **norm** (`Optional[str]`) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** `Tensor`

**Returns** image calculated at scattered Fourier locations.

### 4.3.7 kb\_spmat\_nufft\_adjoint

**kb\_spmat\_nufft\_adjoint** (*data, scaling\_coef, im\_size, grid\_size, interp\_mats, norm=None*)

Kaiser-Bessel adjoint NUFFT with sparse matrix interpolation.

See *KbNufftAdjoint* for an overall description of the adjoint NUFFT.

To calculate the sparse matrix tuple, see *calc\_tensor\_spmatrix()*.

#### Parameters

- **data** (`Tensor`) – Scattered data to be iNUFFT'd to an image.
- **scaling\_coef** (`Tensor`) – Image-domain coefficients to compensate for interpolation errors.

- **im\_size** (`Tensor`) – Size of image with length being the number of dimensions.
- **grid\_size** (`Tensor`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`.
- **interp\_mats** (`Tuple[Tensor, Tensor]`) – 2-tuple of real, imaginary sparse matrices to use for sparse matrix KB interpolation.
- **norm** (`Optional[str]`) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** `Tensor`

**Returns** data transformed to an image.

### 4.3.8 kb\_table\_nufft

**kb\_table\_nufft** (*image, scaling\_coef, im\_size, grid\_size, omega, tables, n\_shift, numpoints, table\_oversamp, offsets, norm=None*)  
Kaiser-Bessel NUFFT with table interpolation.

See *KbNuFFT* for an overall description of the forward NUFFT.

#### Parameters

- **image** (`Tensor`) – Image to be NUFFT'd to scattered data.
- **scaling\_coef** (`Tensor`) – Image-domain coefficients to pre-compensate for interpolation errors.
- **im\_size** (`Tensor`) – Size of image with length being the number of dimensions.
- **grid\_size** (`Tensor`) – Size of grid to use for interpolation, typically 1.25 to 2 times `im_size`.
- **omega** (`Tensor`) – k-space trajectory (in radians/voxel).
- **tables** (`List[Tensor]`) – Interpolation tables (one table for each dimension).
- **n\_shift** (`Tensor`) – Size for fftshift, usually `im_size // 2`.
- **numpoints** (`Tensor`) – Number of neighbors to use for interpolation.
- **table\_oversamp** (`Tensor`) – Table oversampling factor.
- **offsets** (`Tensor`) – A list of offsets, looping over all possible combinations of *numpoints*.
- **norm** (`Optional[str]`) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** `Tensor`

**Returns** image calculated at scattered Fourier locations.

### 4.3.9 kb\_table\_nufft\_adjoint

**kb\_table\_nufft\_adjoint** (*data, scaling\_coef, im\_size, grid\_size, omega, tables, n\_shift, numpoints, table\_oversamp, offsets, norm=None*)

Kaiser-Bessel NUFFT adjoint with table interpolation.

See *KbNufftAdjoint* for an overall description of the adjoint NUFFT.

#### Parameters

- **data** (*Tensor*) – Scattered data to be iNUFFT'd to an image.
- **scaling\_coef** (*Tensor*) – Image-domain coefficients to compensate for interpolation errors.
- **im\_size** (*Tensor*) – Size of image with length being the number of dimensions.
- **grid\_size** (*Tensor*) – Size of grid to use for interpolation, typically 1.25 to 2 times *im\_size*.
- **omega** (*Tensor*) – k-space trajectory (in radians/voxel).
- **tables** (*List[Tensor]*) – Interpolation tables (one table for each dimension).
- **n\_shift** (*Tensor*) – Size for fftshift, usually *im\_size // 2*.
- **numpoints** (*Tensor*) – Number of neighbors to use for interpolation.
- **table\_oversamp** (*Tensor*) – Table oversampling factor.
- **offsets** (*Tensor*) – A list of offsets, looping over all possible combinations of *numpoints*.
- **norm** (*Optional[str]*) – Whether to apply normalization with the FFT operation. Options are "ortho" or None.

**Return type** *Tensor*

**Returns** *data* transformed to an image.





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